

In a recent letter, [1] Gellas *et al.* published results for the spin polarisabilities of the nucleon calculated to next-to-leading order (NLO) in heavy baryon chiral perturbation theory (HB χ PT). Their results differ from those in Refs. [2]. Although this is due to different definitions and not to discrepancies in computing Feynman diagrams, it has caused some confusion in the community. In this comment we show that it is the definition of Ref. [2], and not that of Gellas *et al.*, which should be compared with the values extracted from dispersion relations (DR’s) [4].

The disagreement stems from the standard procedure of excluding the nucleon pole graphs from the definition of nucleon polarisabilities. In particular, in fixed- t dispersion relations each of the six independent Compton amplitudes is split into two parts, a pole piece and a contribution from an integral over the imaginary part of the amplitude above the πN cut; the latter is obtained from photoproduction data. The polarisabilities γ_i are then defined as the lowest terms in the energy expansion of the second (non-pole) piece of the real part of the amplitude. The pole occurs when the intermediate nucleon is on shell. The full pole contribution can be calculated using Dirac spinors with the on-shell γN coupling, and depends only on the nucleon charge, mass and magnetic moment. There are no “off-shell” or “sideways” form-factors involved, and so it is gauge-invariant, Lorentz-covariant, and satisfies the DR’s. The low energy theorems (LETs) for Compton scattering are all pole terms.

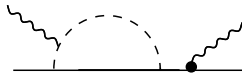


FIG. 1. The disputed diagram.

The difference between the results of Refs. [1,2] is in the treatment of the contribution from Fig. 1. If this is expanded in powers of the photon energy, ω , the leading term is needed to renormalise the bare value of the anomalous magnetic moment in the leading order (LO) pole terms, as might be expected. (Less obviously, many other graphs also have leading terms which contribute to this correction.) The disagreement between Gellas *et al.* and us is in the treatment of the ω^3 terms from Fig. 1. We include them in the polarisabilities, they do not.

It is clear, however, that as the pole terms in the DR’s depend only on the on-shell couplings, any part of the contribution from Fig. 1 (or other loop diagrams) which is not purely a renormalisation of the mass or magnetic moment is not part of the pole term. Thus it should be included in the definition of the polarisabilities if comparison is to be made with the values extracted from DR’s. Indeed by Cauchy’s theorem the polarisability contributions from the amplitude of Fig. 1 are related to an integral over the its imaginary part, which arises from inter-

mediate on-shell πN states—just as in the similar graph where the second photon couples to the nucleon inside rather than outside the pion loop.

For forward scattering, the polarisability γ_0 , can be directly related to the spin-dependent total inelastic cross section via a GDH-style DR. Clearly all diagrams contribute to this. However as the pole contribution beyond the LET vanishes for forward (and backward) angles, γ_0 will be the same whether defined this way or via fixed- t DR’s.

We stress that this is not a problem of relativistic versus heavy-baryon formulations. If however, like Gellas *et al.*, one wishes to propose a new definition of polarisabilities excluding one-particle reducible graphs—polarisabilities which could not then, as we have argued, be directly compared with values extracted from DR’s—then care must be taken that the definition is Lorentz and gauge invariant. In HB χ PT the procedure of Gellas *et al.* violates Lorentz invariance to the order in the expansion in powers of $1/m_N$ to which they are working [3]. Its gauge invariance has also not been demonstrated.

It may be worth stressing here that while HB χ PT will not give the pole at any finite order, it can reproduce the expansion of the amplitude in powers of $1/m_N$. The true pole does not occur at physically realisable values of the energy and momentum transfer, nor is it so close as to make the expansion badly behaved. Thus HB χ PT can calculate the physical amplitude in the vicinity of zero energy without any problems of principle. (Issues of chiral convergence or the effects of the delta are irrelevant to this discussion, though they are important in practice.)

It is true that there are significant discrepancies between the polarizabilities from our NLO HB χ PT calculations and the DR analyses [4]. Since the NLO corrections are of the same magnitude as the LO values however, there is no reason to expect that the series has converged.

Michael C. Birse*, Xiangdong Ji† and Judith A. McGovern*

*Department of Physics and Astronomy, University of Manchester, M13 9PL, U.K.

†Department of Physics, University of Maryland, College Park, MD20742

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- [1] G. C. Gellas, T. R. Hemmert and U.-G. Meißner, Phys. Rev. Lett. **85** 14 (2000).
- [2] X. Ji, C-W. Kao and J. Osborne, Phys. Rev. **D 61** 074003 (2000);
K. B. V. Kumar, J. A. McGovern and M. C. Birse, hep-ph/9909442; Phys. Lett. **B 479** 167 (2000).
- [3] J. A. McGovern, M. C. Birse and K. B. V. Kumar, nucl-th/0007015
- [4] D. Babusci *et al.*, Phys. Rev. **C58** 1013 (1998);
D. Drechsel *et al.*, Phys. Lett. **B 420** 248 (1998);
D. Drechsel *et al.*, Phys. Rev. **C61** 015204 (2000).